Journal of Sound and Vibration 329 (2010) 4583–4587

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Rapid Communication

A generalized flexibility matrix based approach for structural damage detection

Jing Li [a], Baisheng Wu [a,*], Q.C. Zeng [b], C.W. Lim [b]

[a] Department of Mechanics and Engineering Science, School of Mathematics, Jilin University, Changchun 130012, PR China
[b] Department of Building and Construction, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, PR China

ARTICLE INFO

Article history:
Received 7 April 2010
Received in revised form
17 May 2010
Accepted 23 May 2010
Handling Editor: L.G. Tham

ABSTRACT

In this paper, a new structural damage detection approach based on changes in the generalized flexibility matrix is presented. The generalized flexibility matrix is first introduced; its sensitivity and change are then used to detect structural damage location and damage extent. Compared with the original flexibility matrix based approach, the effect of truncating higher-order modes can be considerably reduced in this new approach. Finally, a numerical example for a simply supported beam is used to illustrate the effectiveness of this proposed method.

1. Introduction

It is well known that structural damage results in changes in dynamic characteristics of structures. There have been many attempts which make use of the changes of measured modal parameters to locate and quantify damage. Extensive overviews for methods to detect, locate and characterize damage in structural and mechanical systems by examining changes in measured vibration response can be found in Doebling et al. [1] and Alvandi and Cremona [2]. Among these methods, the approach based on flexibility change has drawn wide attention and it constitutes an important method for structural damage detection [3–8]. The advantage is that the flexibility matrix can be accurately estimated using only a few lower modes and it is very sensitive to damage. Pandey and Biswas [3] first proposed a method for damage detection based on changes in flexibility. Bernal and Gunes put forward the damage locating vector method [4]. They further proposed to use the flexibility proportional matrices method [5] to quantify damage without the use of a model. Qi et al. [6] formulated a flexibility matrix based method for damage identification of truss structures. Cao and Friswell [7] proposed a modal flexibility curvature method for nondestructive damage evaluation. Yang and Liu [8] made use of the eigenparameter decomposition of structural flexibility change to detect structural damage. Although the flexibility identification algorithms have been extensively developed, there are many difficulties inherent in these methods. One of the most difficult problems is that it is very difficult to obtain accurate higher-order modal data.

In this paper, a new structural damage detection method, which uses the generalized flexibility matrix and its sensitivity, is proposed. The generalized flexibility matrix is first introduced and its sensitivity is subsequently used to detect structural damage. Compared with the original flexibility matrix based approach, the effect of truncating higher-order modes can be considerably reduced in this new approach. In specific, this new approach even works with only the first frequency and the corresponding mode shape. Finally, a numerical example for a simply supported beam is used to illustrate the effectiveness of this proposed method.

* Corresponding author.
E-mail addresses: wubs@jlu.edu.cn, bswu@public.cc.jl.cn (B. Wu).

0022-460X/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jsv.2010.05.024

2. Damage detection method

2.1. The generalized flexibility matrix and its sensitivity

In this paper, it is assumed that structural damage does not cause mass variation but only a reduction in the structural stiffness. It is further assumed that the number of degrees of freedom after damage remains unchanged. The damaged, global structural stiffness matrix $\mathbf{K}_d$ is given by

$$\mathbf{K}_d = \mathbf{K}_u - \Delta\mathbf{K} \tag{1}$$

where $\mathbf{K}_u$ is the undamaged $n \times n$ global structural stiffness matrices, n is the number of total degrees of freedom and $\Delta\mathbf{K}$ is the change in the global stiffness matrix.

Referring to the finite element method, a change in the global stiffness matrix can be described as the summation of changes of the elemental stiffness matrices:

$$\Delta\mathbf{K} = \sum_{i=1}^{N} \alpha_i \mathbf{K}_{ui} \tag{2}$$

where N is the number of structural elements, α_i is a scalar denoting the damage extent corresponding to the ith element $(0 \le \alpha_i \le 1)$ and $\mathbf{K}_{ui}$ is the $n \times n$ stiffness matrix of the ith element for the undamaged structure. Due to inevitable measurement noise, the minor damage in global structure is usually undetected. Thus, in this paper, we only consider a damage parameter $\alpha_i \ge 0.05$.

Substituting Eq. (2) into Eq. (1) and differentiating with respect to α_i yield

$$\frac{\partial \mathbf{K}_d}{\partial \alpha_i} = -\mathbf{K}_{ui} \tag{3}$$

Defining $\mathbf{F}_d$ as an $n \times n$ flexibility matrix for the damaged structure, noting $\mathbf{F}_d\mathbf{K}_d = \mathbf{I}$ ($\mathbf{I}$ is the $n \times n$ unit matrix) and differentiating with respect to α_i yield

$$\frac{\partial \mathbf{F}_d}{\partial \alpha_i}\mathbf{K}_d + \mathbf{F}_d \frac{\partial \mathbf{K}_d}{\partial \alpha_i} = 0 \tag{4}$$

Post-multiplying Eq. (4) by $\mathbf{F}_d$ and substituting Eq. (3) into it result in

$$\frac{\partial \mathbf{F}_d}{\partial \alpha_i} = \mathbf{F}_d \mathbf{K}_{ui} \mathbf{F}_d \tag{5}$$

Based on mode shape normalization with respect to mass matrix, a new generalized flexibility matrix will be introduced in this paper. Denoting $\mathbf{M}$ as the $n \times n$ global structural mass matrix, then the generalized flexibility matrix is defined by

$$\mathbf{f}_d^g(\alpha) = \mathbf{F}_d(\mathbf{M}\mathbf{F}_d)^l = \mathbf{\Phi}_d \mathbf{\Lambda}_d^{-1} \mathbf{\Phi}_d^T (\mathbf{M}\mathbf{\Phi}_d \mathbf{\Lambda}_d^{-1} \mathbf{\Phi}_d^T)^l = \mathbf{\Phi}_d \mathbf{\Lambda}_d^{-1-l} \mathbf{\Phi}_d^T, \quad l = 0,1,2,\ldots \tag{6}$$

where $\mathbf{\Phi}_d$ and $\mathbf{\Lambda}_d$ are the mode shape matrix and diagonal matrix of natural frequency squared, respectively, for the damaged structure. From Eq. (6), it can be drawn that a larger l causes reduced contribution of higher-order mode. For l=0, Eq. (6) reduces to the original flexibility matrix, namely $\mathbf{F}_d = \mathbf{\Phi}_d \mathbf{\Lambda}_d^{-1} \mathbf{\Phi}_d^T$. In this paper, only l=1 is considered.

For l=1, the generalized flexibility matrix in Eq. (6) becomes

$$\mathbf{f}_d^g(\alpha) = \mathbf{F}_d \mathbf{M} \mathbf{F}_d = \mathbf{\Phi}_d \mathbf{\Lambda}_d^{-1} \mathbf{\Phi}_d^T \mathbf{M} \mathbf{\Phi}_d \mathbf{\Lambda}_d^{-1} \mathbf{\Phi}_d^T = \mathbf{\Phi}_d \mathbf{\Lambda}_d^{-2} \mathbf{\Phi}_d^T \tag{7}$$

Differentiating Eq. (7) with respect to α_i leads to

$$\frac{\partial \mathbf{f}_d^g}{\partial \alpha_i} = \frac{\partial \mathbf{F}_d}{\partial \alpha_i}\mathbf{M}\mathbf{F}_d + \mathbf{F}_d \mathbf{M} \frac{\partial \mathbf{F}_d}{\partial \alpha_i} \tag{8}$$

Substituting Eq. (5) into Eq. (8) and setting $\alpha_i = 0$ ($i = 1,\ldots,N$) yield

$$\left.\frac{\partial \mathbf{f}_d^g}{\partial \alpha_i}\right|_{\alpha_i = 0} = \mathbf{F}_u \mathbf{K}_{ui} \mathbf{F}_u \mathbf{M} \mathbf{F}_u + \mathbf{F}_u \mathbf{M} \mathbf{F}_u \mathbf{K}_{ui} \mathbf{F}_u \tag{9}$$

where $\mathbf{F}_u$ is the $n \times n$ flexibility matrix for the undamaged structure. Thus, sensitivity of the generalized flexibility matrix is determined.

2.2. Structural damage detection method

In this paper, it is assumed that the measured mode shape vector has the same dimension as the analytical mode shape vector. This assumption allows focus of attention on the quality of the proposed method.

Making use of Taylor's series expansion for $\mathbf{f}_d^g$ at $\alpha_i = 0\ (i = 1,\ldots,N)$, the first-order approximation to the generalized flexibility matrix $\mathbf{f}_d^g$ can be expressed as

$$\mathbf{f}_d^g \approx \mathbf{f}_u^g + \sum_{i=1}^{N} \alpha_i \frac{\partial \mathbf{f}_d^g}{\partial \alpha_i}\bigg|_{\alpha_i = 0} \tag{10}$$

Substituting Eq. (9) into Eq. (10) and rearranging yield

$$\Delta\mathbf{f} = \mathbf{f}_d^g - \mathbf{f}_u^g \approx \sum_{i=1}^{N} \alpha_i (\mathbf{F}_u \mathbf{K}_{ui} \mathbf{F}_u \mathbf{M} \mathbf{F}_u + \mathbf{F}_u \mathbf{M} \mathbf{F}_u \mathbf{K}_{ui} \mathbf{F}_u) \tag{11}$$

where $\mathbf{f}_u^g = \mathbf{F}_u \mathbf{M} \mathbf{F}_u$ and $\Delta\mathbf{f}$ is the change of the generalized flexibility matrix.

The generalized flexibility matrix can be approximately determined by using only a few of the lower frequency modes. From Eq. (7), $\Delta\mathbf{f}$ can be approximately expressed as

$$\Delta\mathbf{f} = \mathbf{F}_d \mathbf{M} \mathbf{F}_d - \mathbf{F}_u \mathbf{M} \mathbf{F}_u \approx \sum_{j=1}^{m} \frac{1}{\omega_{dj}^4} \mathbf{\Phi}_{dj} \mathbf{\Phi}_{dj}^T - \sum_{j=1}^{m} \frac{1}{\omega_{uj}^4} \mathbf{\Phi}_{uj} \mathbf{\Phi}_{uj}^T \tag{12}$$

where ω_{dj} and $\mathbf{\Phi}_{dj}$ are the jth frequency and the corresponding jth mode shape for the damaged structure, ω_{uj} and $\mathbf{\Phi}_{uj}$ are the jth frequency and the corresponding jth mode shape for the undamaged structure, m is the number of measured modes. Then, the damage parameters can be calculated by manipulating Eqs. (11) and (12) into a system of linear equations with respect to $\alpha_i\ (i = 1,\ldots,N)$. Solving this system of linear equations using the least squares method, both damage location and damage extent can be determined.

For the original flexibility matrix based approach, the change of flexibility matrix can be expressed as

$$\Delta\mathbf{F} = \mathbf{F}_d - \mathbf{F}_u \approx \sum_{j=1}^{m} \frac{1}{\omega_{dj}^2} \mathbf{\Phi}_{dj} \mathbf{\Phi}_{dj}^T - \sum_{j=1}^{m} \frac{1}{\omega_{uj}^2} \mathbf{\Phi}_{uj} \mathbf{\Phi}_{uj}^T \tag{13}$$

In Eq. (12), the jth term of $\Delta\mathbf{f}$ is proportional to $1/\omega_{dj}^4$, while the jth term of $\Delta\mathbf{F}$ is only proportional to $1/\omega_{dj}^2$. Therefore, compared with the original flexibility matrix based approach, the effect of the truncating higher-order modes can be considerably reduced in the proposed generalized flexibility method.

3. Numerical example

A simply supported beam with rectangular cross section shown in Fig. 1 is used to validate the proposed method. The material and geometric constants are as follows: Young's modulus $E = 200\,\text{GPa}$, density $\rho = 7800\,\text{kg}/m^3$, length l=1 m,

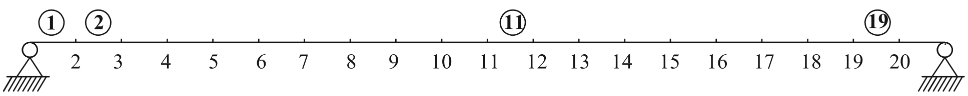

Fig. 1. A simply supported beam.

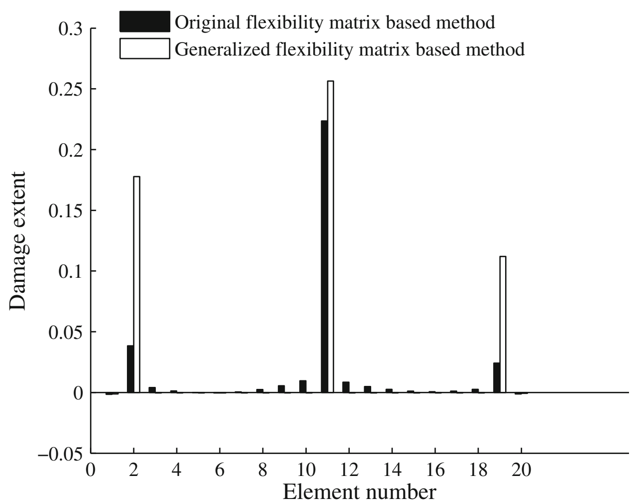

Fig. 2. Damage detection for case I without noise by using only the first mode.

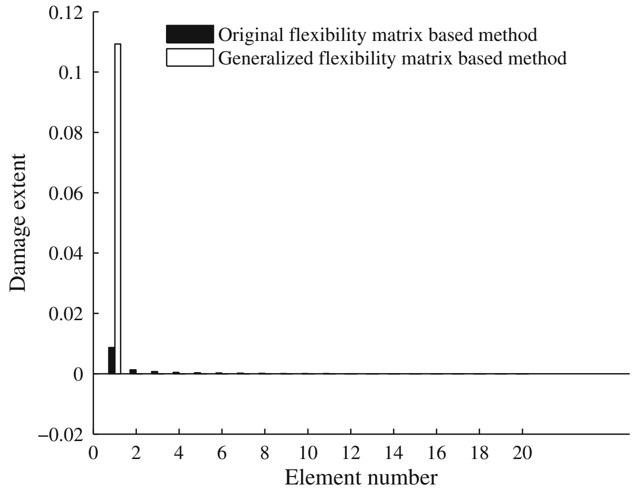

Fig. 3. Damage detection for case II without noise by using only the first mode.

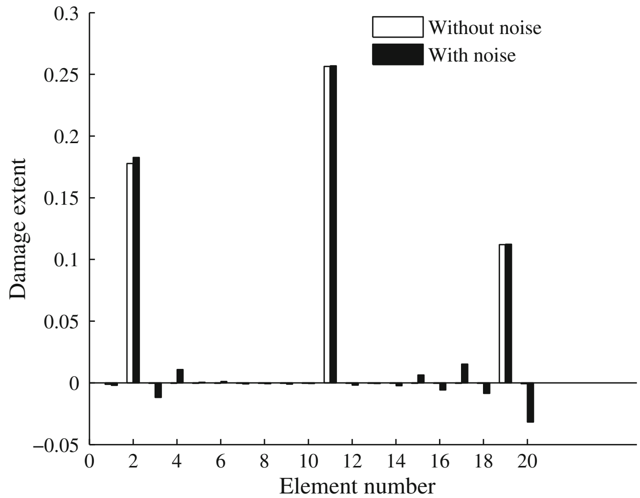

Fig. 4. Damage detection for case I by using only the first mode.

cross sectional area $A=10\,\text{mm}\times 50\,\text{mm}$ and second moment of area $I_z=4.17\times 10^{-9}\,\text{m}^4$. The total numbers of elements and nodes are 20 and 21, respectively. Here, all degrees of freedom along the x-axis are ignored. Two damage cases are studied in this example: case I, elements 2, 11 and 19 are damaged simultaneously with stiffness losses of 15%, 20% and 10%, respectively; and case II, only element 1 is damaged with a stiffness loss of 10%.

Solving Eqs. (11) and (12) with $m=1$, the damage locations and damage extents can be determined using only the first frequency and the corresponding mode shape. For comparison, the damage locations and damage extents are also calculated using the original flexibility matrix based approach with $m=1$. The results for cases I and II are shown in Figs. 2 and 3. Fig. 2 demonstrates that for case I, the original flexibility matrix based approach is unable to detect damages in elements 2 and 19. Note that a damage parameter $\alpha_i\geq 0.05$ is considered in this paper. In contrast, the damage location can be exactly detected by using the proposed generalized flexibility matrix method with only the first frequency and mode shape. The damage extents detected are 0.1777, 0.2564 and 0.1120 for elements 2, 11 and 19, respectively. For case II, Fig. 3 indicates that the damage in element 1 cannot be detected by the original flexibility matrix based approach while, in contrast, it can be accurately detected by using the generalized flexibility matrix method with only the first frequency and mode shape. The damage extent in element 1 is 0.1093 which is very close to the actual damage extent 0.10. We also assume the frequencies and mode shapes are contaminated with 1% and 5% random noises, respectively [9,10]. Figs. 4 and 5 are the results for cases I and II using generalized flexibility matrix method without noise and with noise, respectively.

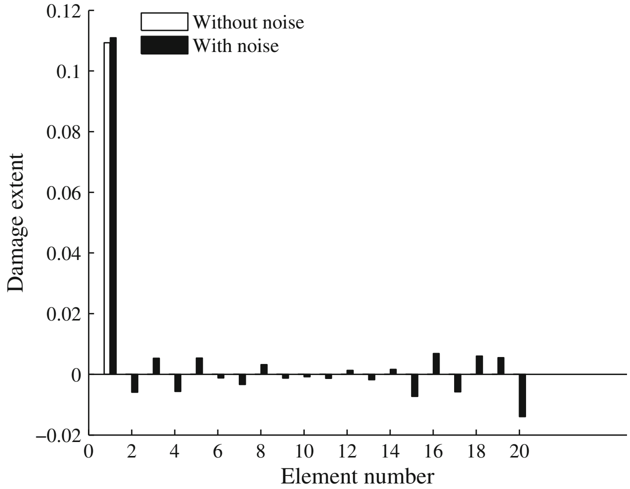

Fig. 5. Damage detection for case II by using only the first mode.

From the results above, it is clear that the proposed new method is very efficient with only the first frequency and its corresponding mode shape. Accurate detection can also be determined even with noise contamination in the measured data.

4. Conclusions

In this paper, a new efficient method of structural damage detection has been developed. The generalized flexibility matrix is first introduced; its sensitivity and change are then used to detect the structural damage location and the damage extent. Compared with the original flexibility matrix based approach, the effect of truncating higher-order modes can be considerably reduced in the new method. The efficiency of the proposed method has been demonstrated using a simply supported beam in the numerical example with only the lowest measured mode considered. From the numerical results, it can be concluded that the proposed method is very efficient especially in determining the damage locations regardless of the existence of a single damage or multiple damages. Numerical simulations show that the proposed method works well even for data with noise.

References

[1] S.W. Doebling, C.R. Farrar, M.B. Prime, A summary review of vibration-based damage identification methods, *Shock and Vibration Digest* 30 (2) (1998) 91–105.
[2] A. Alvandi, C. Cremona, Assessment of vibration-based damage identification techniques, *Journal of Sound and Vibration* 292 (2006) 179–202.
[3] A.K. Pandey, M. Biswas, Damage detection in structures using changes in flexibility, *Journal of Sound and Vibration* 169 (1) (1994) 3–17.
[4] D. Bernal, B. Gunes, A flexibility based approach for the localization and quantification of damage: application in a benchmark structure, *Proceedings of the Third International Workshop in Structural Health Monitoring*, Stanford, CA, 2001, pp. 680–689.
[5] D. Bernal, B. Gunes, Flexibility based approach for damage characterization: benchmark application, *Journal of Engineering Mechanics* 130 (1) (2004) 61–70.
[6] B.H. Qi, R.F. Wu, X.H. Cai, G.H. Li, A flexibility matrix method for damage identification of truss structure, *Chinese Journal of Computational Mechanics* 18 (1) (2001) 42–47.
[7] H. Cao, M.I. Friswell, Nondestructive damage evaluation indicator based on modal flexibility curvature, *Chinese Journal of Engineering Mechanics* 23 (4) (2006) 33–38.
[8] Q.W. Yang, J.K. Liu, Damage identification by the eigenparameter decomposition of structural flexibility change, *International Journal for Numerical Methods in Engineering* 78 (4) (2009) 444–459.
[9] P.L. Liu, Identification and damage detection of trusses using modal data, *Journal of Structural Engineering* 121 (4) (1995) 599–608.
[10] L. Papadopoulos, E. Garcia, Structural damage identification: a probabilistic approach, *AIAA Journal* 36 (11) (1998) 2137–2145.